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Improved Predictive Modeling Using Bayesian Additive Regression Trees(BART) For Wood Composite Products

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Abstract: *This study presents an ensemble of predictive models with a focus on the predictive capabilities of Bayesian Additive Regression Trees (BART). Predictions are made for Modulus of Rupture (MOR) andTensileStrength(IB or Internal Bond) from a wood compositesmanufacturing process for three product types. Given the large number of predictor variables from the process, variable preselection was used prior to model development. Several regression methods including multiple linear regression, partial least squares regression, neural networks, regression trees, boosted trees, and bootstrap forest are compared with BART.BART had the best predictiveperformance in validation unanimously for bothMORand IBfor all three products examined. Bootstrap forest validation results were very similar to BART for one of the products. BART validation results of MORwere promisingfor the nominal product type of 19.05 mm with an* $r = 0.89$ *for 10-fold crossvalidationwith root mean square error of prediction (NRMSEP) of 10.26%. BART validation results for IBhad anaverager* $= 0.84$ *for10-fold cross-validation with aNRMSEP = 10.82%. The high predictive ability of BARTmay be useful for manufacturers and researchers in applying analytical techniques for process improvement leading to less rework (order reruns due to failing properties) and reject. Predictive modeling techniques like the ones explored in this study may be very important to companies seeking competitive advantage in today's business world that is focused on advanced analytics and data mining.*

Keywords-*Bayesian Additive Regression Trees (BART), manufacturing, predictive models*

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I. Introduction

Today's business world is using advanced analytics and data mining for competitive advantage. Wood composites industries exist in a business climate that is highly competitive where lowering cost of manufactured product is a key element to success in commodity-based products. Predictive modeling is the basis for data mining and induction. Predictive models that are accurate may help manufacturers reduce rework (or reruns of schedule due to failing properties) and scrap; and may also help manufacturers diagnose unknown sources of variation from the process ([1], [2]). Variation creates significant costs for manufacturers in that variation influences targets for weight, thickness, drying, etc., *i.e.,* the more variation in the process, the higher the operational targets and the greater the cost ([3]).

There is a plethora of literature over the last two decades on predictive modeling for forest products from industrial processes, see citations by [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20]. In most of the aforementioned literature, partial least squares (PLS) regression or an adaptation of PLS,were the best predictive modelingtechniques. The use of Bayesian Adaptive Regression Trees (BART) was explored in this study for predictive modeling of the module of rupture (MOR) and internal bond (IB) from a wood composites manufacturing process. Predictive models from BART were compared with six other modeling techniques. This study advances the literature by documenting the predictive capabilities of BART for wood composites.

Wood composites manufacturing processeshave a large number of interdependent process variables with complex interrelationships; and these process variables influence final board properties, *e.g.,* wood chip dimensions, fiber dimensions, fiber-resin formations, mat-forming consistency, line speed, press closing characteristics, fiber moisture, etc. Given that a process database may contain hundreds of process variables that have some correlation with final board properties, variable pre-selection before modeling is a useful technique to reduce dimensionalityand improve model predictive performance.Several approaches for variable preselection exist, such as iterative variable selection ([21]), uninformative variable elimination ([22], [23]), and iterative predictor weighting ([24]). Recently, it has been shown that Genetic Algorithms(GA), even though computationally intensive, is a useful variable preselection technique ([20], [25],[26],[27]). Variable preselection improvedthe predictive modeling performance in the study by [13].

The objective of this study was to compare the predictive capabilities for seven regression techniques, predicting theMOR and IB for a particleboard manufacturing process. Even though this paper focuses on particleboardprocess as a case study, these same approachesmay be applied to other wood products and other industries. The regression techniques explored were: multiple linear regression(MLR), partial least squares (PLS), neural networks (NN), regression trees (RT), boosted trees (BRT), bootstrap forests (BSF), and Bayesian additive regression trees (BART).

II. Methods

2.1 Database

Destructive testing to determine MOR and IB are the standard test methods for defining the strength quality of particleboard during manufacturing. Strength quality attributes of particleboard are unknown at the time of manufacture.Destructive test samples are generally taken from the production line at one or two hour time-intervals, and sometimes at product type changes. The dataset for this study consisted of 4,307 records from March 2009 and June 2010.There were 189 possible predictor variables from sensors on the production linethat were aligned properly '*in-time*' with both MOR and IB destructive tests. Some of the predictor variables explored wererelated to line speed, forming, weight, fiber moisture, mat temperature, press temperature by zone, and press pressure by zone, etc. Models were developed for three main product types: 12.70 *mm*; 15.88 *mm*; and 19.05 *mm*. The record length by product type were: n =166 (12.70 *mm*), n = 184 (15.88 *mm*), and n = 487(19.05 *mm*).

2.2 Regression Methods

Mixed stepwise regressionand GA were explored for variable preselection. Several training models were built using the seven regression after variable preselection. Mixed stepwise regression had better performance than GA as a preselection technique for predictions in validation across all of the seven model techniques studied. Given that a complete review of regression methods related to MLR, PLS, NN would be quite extensive, and these methods are well documented in the literature, the readers are referred to several helpful citations for a thorough review ([28], [29], [30]). A review of'*tree-based*' regression methods may be more informativefor the purpose of this paper since its focus is on the predictive capabilities of BART.

Regression tree methods partition the data space into homogeneous sub-regions using a variety of techniques. Regression tree models have the advantage of high explanatory value and stability. Regression trees (RT) identify a hierarchy of interactions, and some unknown interactions, that allow for an improved understanding of the interrelationships effecting the dependent variable. RTs are quite helpful in analyzing high-dimensional datasets and have few model assumptions.Even though RTs have high explanatory value and stability, such methods have poor predictive performance from the segmentation of the database space into homogeneous sub regions. This leads to a large generalized error of predictions for each subspace.

*'Tree Boosting'*relies on the philosophy that a small number of simple trees of weak learners that are combined as one model outperforms the predictions of one large RT ([31], [32]). *'Boosting'*builds small one to three node trees sequentially as an ensemble to improve predictions.The result grows new trees by accommodating observations that the existing ensemble predicts poorly, *i.e.*,the sum-of-trees improves predictive performance of the final BRT model. ([33]). As stated by Schapire [32] and Elith *et al.* [33], BRT is a model to enhance the model accuracy and the key step in *'Boosting'* is to consecutively apply the algorithm to constantly modified data, *i.e.*, it minimizes the loss function through adding a regression tree in each iteration step ([33]).

BSF provides a useful method for estimating other percentiles of the data. Most regression tree methods focus on the estimating the mean of the data space. The essentialidea of the BSF is that the *'bootstrap distribution'* is approximated not assuming a prior distribution such as the Gaussian The BSF approach was helpful in estimating the interval of lower percentiles of wood compositesin a study by Edwards *et al.*[15].

Bayesian additive regression trees (BART) use the same approach asBRT in that trees in the ensemble are grown sequentially to reduce the generalized error of prediction. As Chipman [34] noted, "*the entire model is regularized so that no one tree dominates the prediction of the response surface*." Unlike BRTs, tree growing assumes that the parameters of the probability density function of Y are estimated under Bayesian framework ([34]).

For example, if we specify that the BART model should have 50 trees, a *posterior* sample of the 50 trees is createdspecifiedby the observations and*prior*distribution. As Hill [35] noted, "*the result is a highly flexible, data-responsive ensemble method, which produces measures of uncertainty in the very process of finding a sum-of-trees that accurately reproduces a given outcome surface*,"*i.e.,* improved predictive performance.

BART is a sum-of-trees ensemble which has a greater ability to capture interactions and non-linearity and additive effects among variables ([36]). Meanwhile, the estimation approach of BART relies on the Bayesian probability model. The specific expression of BART model is presented as:

$$Y = f(X) + \varepsilon \approx T_1^Z(X) + T_2^Z(X) + \cdots + T_m^Z(X) + \varepsilon \quad [3]$$

$$\varepsilon \sim N_n(0, \sigma^2 I_n)$$

where Y is the $n \times 1$vector of response variables, X is the $n \times p$ matrix in which the predictor column joined and ε is the residues. Suppose we have m district regression trees and each has a tree structure denoted by$T$ and the terminal nodes represented by Z. Thus, T^{Z} is the tree composing of structure and parameters. BART composes numerous *priors* for the structure and the leaf parameters of the trees ([34],[36]).

2.3 Model Validation

Ten-fold cross validation was employed to determine the predictive capabilities of the regression techniques. The 80% training and 20% testing (or validation) rule was applied for each product type. The average of the root mean squared error for prediction (RMSEP) was calculated for the 10-fold cross validations to compare the predictive capabilities. RMSEP is:

$$\mathrm{RMSEP}_j = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} for j = 1, \ldots, 10 \qquad [4]$$

where y_i is the actual value, $\widehat{y_i}$ is the predicted value, and n is the total number of records in the validation data set.The normalized root mean squared error for prediction (NRMSEP)was also calculated as:

$$\mathrm{NRMSEP} = \frac{RMSEP}{y_{max} - y_{min}} \qquad [5]$$

where $y_{max} - y_{min}$ represents the data range of the responsible variable in the validation data set. NRMSEP was used given the differences in scale for MOR and IB. The purpose for reporting NRMSEP was to compare the predictive performance of each model for both MOR and IB regardless of scale.

III. Results and Discussion

The results showed that 34 predictors were significantly correlated with MOR, while 31 variables were significantly correlated with IB. Predictors for MOR including process variables related to line mat weight, density, as well as press pressures and temperatures etc.Given the terms of a confidentiality agreement between the manufacturer and university, specific details of the names of the predictors were masked. These findings as related to the type of predictor variables are consistent with other research, seeAndré and Young (2013),Wong *et al.* (1999), Jin *et al.* (2009).The validation results are summarized in Tables 2-7. XY scatterplots of predicted $(\widehat{y})$versus actual (y) are presented in Figs 1-6for the first three best predictive models.The key difference from previous research is that BART models were not explored in the context of predictive performance; and that BART has improved predictive performance relative to previous research for large data sets (recall the dataset used had n = 4,307). This study's contribution research is in the advancement of analytics and data miningmethods for improved decision making by practitioners.

3.1 Product Type 12.70mm Thickness

The validation statistics of seven models for 12.70 *mm* thickness of MOR and IB are given in Tables 2 and 3. Comparing the RMSEPof MOR for the seven regression techniques (MLR, PLS, NN, DT, BT, BF, and BART); the BART model was the best model to predict MOR across 10-fold cross validation. The BART model for MOR had an$r$ = 0.85(**Fig. 1a**) for the best validation dataset,with an average RMSEP =693.19*kPa* and an average NRMSEP = 12.82% across 10-fold cross validation. BRT was a close performer to BART with an$r$ = 0.83 (**Fig. 1b**) for the best validation dataset, with an average RMSEP =734.28*kPa* and an average NRMSEP = 13.58% across 10-fold cross validation. BSF was the third best model using this criteria of predictive performance metrics. XY scatter plots of actual MOR and predicted MOR for the best validation datasetfor BART, BRT, and BSF are given in**Fig. 1**. The *XY* scatter plots indicate good predictive performance across the data range for these three algorithms.

Similar model ranking results were observed for IB, but predicting IB was more successful than predicting MOR given the improved predictive metrics for this case study (Table 3). BART had an r = 0.82 (**Fig. 2a**) for the best validation dataset, with a RMSEP =55.78*kPa*, and a NRMSERP = 10.51%. BRT was the second best predictive model with an r = 0.77 (**Fig. 2b**) for the best validation dataset, with a RMSEP =63.45*kPa*, and a

NRMSEP = 11.95%. BSF had an r = 0.73 (**Fig. 2c**) for the best validation dataset, with a RMSEP =67.94*kPa* and NRMSEP = 12.80%. The *XY* scatter plots in **Fig. 2a, 2b, 2c** of actual and predicted IB highlight the predictive performance across the data range.

3.2Product Type 15.88 mm Thickness

Validation resultsfor the seven regression models for15.88 *mm*thickness are summarized in Tables 4 and 5. Similar results were observed for 15.88 *mm* as was the case for 12.70 *mm*. BART has the best predictive results across 10-fold cross validation for MOR. BART had an r =0.83 (**Fig. 3a**) for the best validation dataset, with an average RMSEP = 645.25*kPa*and an averageNRMSEP = 11.89% across 10-fold cross validation. Validation results for BRT were slightly had an r = 0.77 (*Fig. 3b*) for the best validation dataset, with an average RMSEP = 753.96*kPa* and an average NRMNSEP = 13.89% across 10-fold cross validation. BSF had an r = 0.78 (**Fig. 3c**) for the best validation dataset, with an average RMSEP = 764.49 *kPa* RMSEP, and an average NRMSEP = 14.09% across 10-fold cross validation. The three *XY* scatter plots for this product revealed no apparent bias in predictions (**Fig. 3a, 3b, 3c**).

BART also had the best predictive performance for IB with an r = 0.86 (Fig. 4a) for the best validation dataset, with an average RMSEP = 50.0*kPa*,an average NRMSEP = 10.82% across 10-fold cross validation. BRT had an r = 0.82 (**Fig. 4b**) for the best validation dataset, with an average RMSEP =53.92*kPa*, and NRMSEP = 11.67% across 10-fold cross validation. BSF had an r = 0.79 (**Fig. 4c**) for the best validation dataset, RMSEP =59.63*kPa*, and NRMSEP =12.91% across 10-fold cross validation. There was no apparent bias in predictions in validation for BART, BRT, and BSF (**Fig. 4**).

3.3 Product Type 19.05 mm Thickness

Validation statistics for each regression model for19.05 *mm* thickness MOR and IB are summarized in Tables 6 and 7. BART was best predictive regression technique in validation for MOR in eight of the ten cross validation datasets. BART had a RMSEP = 579.16*kPa*, NRMSEP = 10.26% for the 10-fold cross validations,with an r = 0.89 (**Fig. 5a**)for the best validation data set. BSF predictions for MOR in validation had an r = 0.87 (**Fig. 5c**) or the best validation data set, average RMSEP =614.10*kPa*, and an average NRMSEP = 10.88% for the 10-fold cross validations. BRT predictions for MOR had an r = 0.85 (**Fig. 5b**) for the best validation data set, with an average RMSEP = 655.01*kPa*, average NRMNSEP = 11.60% for the 10-fold cross validations. *XY*scatter plots of the actual MOR versus predicted MOR did not reveal any bias(**Fig. 5a, 5b, 5c**).

Results were somewhat different for IB for the 19.05 *mm* product. BSF had better RMSEP and NRMSEP in four of the 10-fold cross validations, and BRT had the best RMSEP and NRMSEP in one case. BART had the lowest average and standard deviation for RMSEP and NRMSEP across the 10-fold cross validations. BART had an r = 0.84 (**Fig. 6a**) for the best validation data set, with an average RMSEP = 51.92*kPa*, an average NRMSERP = 8.96% for the 10-fold cross validations. BSF had an$r$ = 0.83 (**Fig. 6**c) for the best validation data set, average RMSEP = 51.94*kPa*, and an average NRMSEP = 8.97% for the 10-fold cross validations. BRT had an r = 0.79 (**Fig. 6b**) for the best validation data set, average RMSEP = 57.04 *kPa*, and an average NRMSEP = 9.85% for the 10-fold cross validations. The improved performance of BSF for IB for the 19.05 *mm* product may be the result of the bootstrap distributions instead of the priors which are part of BART.

Based on the validation results of RMSEP and NRMSEP;BART, BRT, and BSF outperformed the regression techniques MLR, PLS, NN, and DT which are common techniques in prior research. André and Young (2013)found that PLS was a good predictive technique, but '*tree-based*' methods were not explored in their research and their dataset size was smaller. The good performance of BART modelsleads credence to the use of Bayesian theory in capturing the interactive effectsand additive effects that may be common to the strength properties of wood composites.

IV. Conclusions

This study demonstrated the predictive capabilities of '*tree-based*' regression methods in the context of Bayesian theory for wood composites quality attributes MOR and IB. Among the seven different regression techniques explored in this study, Bayesian Adaptive Regression Trees (BART)hadpredominately the best predictiveperformance across 10-fold cross validations based on the minimum RMSEP and NRMSEP for three different thickness product types. The strong predictive capabilities of BART modelsmay be beneficial to manufacturers in predicting failures, thus reducing reruns of order-file because of failing properties. These techniques may also help manufacturers diagnose sources of variation in the process that were not previously feasible usingonly non-analytical methods. Given that HCHO emissions from wood composite panels is a significant issue for this industry, future research should explore predicting HCHO emissions from wood composite panels using '*tree- based*' methods.

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Figures and Tables

Table 1. Descriptive statistics for of MOR and IB for three product thicknesses.

Thickness	Property	*N*	Mean (kPa)	Minimum	Maximum	Standard deviation	CV (%)
12.70 *mm*	MOR	166	13313.71	10383.51	15789.00	1045.45	7.85
	IB	166	640.96	372.32	903.21	91.58	14.29
15.88 *mm*	MOR	184	13122.25	10328.35	15754.53	998.98	7.61
	IB	184	620.08	427.48	889.42	81.12	13.08
19.05 *mm*	MOR	487	13107.35	10183.56	15830.37	941.62	7.18
	IB	487	595.72	344.74	923.90	80.38	13.49

Table 2: RMSEPs from different models for standardized dataset with 12.70 *mm* MOR (*kPa*) as the response

	MLR		PLS		NN		DT		BRT		BSF		**BART**	
	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP
1	753.00	13.93%	939.67	17.38%	828.68	15.33%	968.74	17.92%	778.15	14.40%	778.83	14.41%	**684.28**	**12.66%**
2	749.73	13.87%	762.09	14.10%	992.77	18.37%	1031.12	19.08%	**596.89**	**11.04%**	747.12	13.82%	681.20	12.60%
3	832.96	15.41%	862.44	15.95%	943.90	17.46%	911.15	16.86%	766.65	14.18%	718.06	13.28%	**667.04**	**12.34%**
4	771.34	14.27%	936.18	17.32%	850.28	15.73%	1054.83	19.51%	738.76	13.67%	760.40	14.07%	**684.32**	**12.66%**
5	812.23	15.03%	868.84	16.07%	871.22	16.12%	1018.15	18.84%	790.61	14.63%	728.77	13.48%	**725.09**	**13.41%**
6	750.61	13.89%	751.59	13.90%	954.02	17.65%	983.61	18.20%	761.10	14.08%	725.54	13.42%	**689.10**	**12.75%**
7	800.70	14.81%	864.58	15.99%	924.43	17.10%	985.18	18.23%	**743.69**	**13.76%**	790.59	14.63%	745.94	13.80%
8	746.71	13.81%	764.95	14.15%	916.56	16.96%	990.07	18.32%	734.27	13.58%	765.59	14.16%	**674.16**	**12.47%**
9	745.83	13.80%	928.64	17.18%	828.69	15.33%	915.37	16.93%	729.65	13.50%	723.73	13.39%	**693.58**	**12.83%**
10	773.01	14.30%	774.32	14.32%	1001.27	18.52%	955.51	17.68%	703.01	13.01%	814.37	15.07%	**687.14**	**12.71%**
Average	773.61	14.31%	845.33	15.64%	911.18	16.86%	981.37	18.16%	734.28	13.58%	755.30	13.97%	**693.19**	**12.82%**
Std dev	31.22		76.40		63.94		46.47		54.62		32.42		**24.04**	

Table 3. RMSEPs from different models for standardized dataset with 12.70 *mm* IB (*kPa*) as the response.

	MLR		PLS		NN		DT		BRT		BSF		**BART**	
	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP
1	70.12	13.21%	67.56	12.73%	76.20	14.35%	76.29	14.37%	59.14	11.14%	67.71	12.75%	**54.55**	**10.28%**
2	63.27	11.92%	67.56	12.73%	68.28	12.86%	80.21	15.11%	71.19	13.41%	70.35	13.25%	**54.28**	**10.22%**
3	59.76	11.26%	63.90	12.04%	71.15	13.40%	85.69	16.14%	70.36	13.25%	68.42	12.89%	**54.86**	**10.33%**
4	65.74	12.38%	69.34	13.06%	70.77	13.33%	85.60	16.12%	62.89	11.85%	66.54	12.53%	**56.36**	**10.62%**
5	66.06	12.44%	65.28	12.30%	78.07	14.71%	86.39	16.27%	69.74	13.14%	65.44	12.33%	**52.50**	**9.89%**
6	75.73	14.26%	73.92	13.92%	77.17	14.54%	83.84	15.79%	57.61	10.85%	66.81	12.58%	**53.16**	**10.01%**
7	160.49	30.23%	77.98	14.69%	74.57	14.05%	83.52	15.73%	**59.18**	**11.15%**	66.91	12.60%	68.65	12.93%
8	61.98	11.67%	78.94	14.87%	91.41	17.22%	86.21	16.24%	60.14	11.33%	72.49	13.65%	**54.63**	**10.29%**
9	66.59	12.54%	81.47	15.35%	90.67	17.08%	85.34	16.07%	61.75	11.63%	69.16	13.03%	**54.46**	**10.26%**
10	63.15	11.89%	72.10	13.58%	68.43	12.89%	85.14	16.04%	62.50	11.77%	65.58	12.35%	**54.38**	**10.24%**
Average	75.29	14.18%	71.81	13.53%	76.67	14.44%	83.82	15.79%	63.45	11.95%	67.94	12.80%	**55.78**	**10.51%**
Std dev	30.27		6.09		8.32		**3.21**		5.09		**2.22**		4.63	

Table 4. RMSEPs from different models for standardized dataset with *15.88 mm* MOR (*kPa*) as the response.

	MLR		PLS		NN		DT		BRT		BSF		**BART**	
	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP
1	785.18	14.47%	902.85	16.64%	884.59	16.30%	986.75	18.18%	854.14	15.74%	772.97	14.25%	**640.12**	**11.80%**
2	775.45	14.29%	884.34	16.30%	953.87	17.58%	927.51	17.09%	826.75	15.24%	722.86	13.32%	**609.41**	**11.23%**
3	759.69	14.00%	890.61	16.41%	750.99	13.84%	821.33	15.14%	837.23	15.43%	867.23	15.98%	**664.18**	**12.24%**
4	810.76	14.94%	941.96	17.36%	819.57	15.10%	918.97	16.94%	850.44	15.67%	765.70	14.11%	**688.50**	**12.69%**
5	845.95	15.59%	918.88	16.93%	842.46	15.53%	953.37	17.57%	729.50	13.44%	682.33	12.57%	**609.73**	**11.24%**
6	799.27	14.73%	805.3	14.84%	812.79	14.98%	849.90	15.66%	**546.79**	**10.08%**	656.12	12.09%	655.38	12.08%
7	766.36	14.12%	773.46	14.25%	928.76	17.12%	907.50	16.72%	**606.16**	**11.17%**	646.93	11.92%	673.02	12.40%
8	798.95	14.72%	888.8	16.38%	875.01	16.13%	715.46	13.19%	816.30	15.04%	820.48	15.12%	**649.04**	**11.96%**
9	814.45	15.01%	873.27	16.09%	859.72	15.84%	729.85	13.45%	737.88	13.60%	974.17	17.95%	**651.64**	**12.01%**
10	860.6	15.86%	904.19	16.66%	787.68	14.52%	884.01	16.29%	734.39	13.53%	736.06	13.56%	**611.47**	**11.27%**
Average	801.67	14.77%	878.37	16.19%	851.54	15.69%	869.47	16.02%	753.96	13.89%	764.49	14.09%	**645.25**	**11.89%**
Std dev	32.76		51.23		62.31		90.79		106.18		101.43		**27.63**	

Table 5. RMSEPs from different models for standardized dataset with 15.88 *mm* IB (kPa) as the response.

	MLR		PLS		NN		DT		BRT		BSF		**BART**	
	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP
1	65.00	14.07%	65.48	14.17%	62.14	13.45%	73.47	15.90%	50.18	10.86%	58.25	12.61%	**45.41**	**9.83%**
2	66.64	14.43%	71.37	15.45%	71.91	15.57%	67.82	14.68%	60.44	13.08%	58.90	12.75%	**47.06**	**10.19%**
3	67.51	14.61%	67.19	14.54%	66.71	14.44%	73.45	15.90%	54.40	11.78%	60.76	13.15%	**50.97**	**11.03%**
4	65.08	14.09%	70.23	15.20%	62.33	13.49%	72.37	15.67%	**48.14**	**10.42%**	61.37	13.29%	51.00	11.04%
5	64.89	14.05%	57.99	12.55%	66.04	14.30%	70.28	15.21%	**53.45**	**11.57%**	57.19	12.38%	54.91	11.89%
6	67.41	14.59%	68.94	14.92%	66.00	14.29%	71.52	15.48%	59.57	12.90%	53.92	11.67%	**49.45**	**10.70%**
7	64.76	14.02%	69.84	15.12%	75.18	16.27%	70.62	15.29%	51.46	11.14%	63.08	13.66%	**49.33**	**10.68%**
8	68.29	14.78%	70.28	15.21%	64.69	14.00%	71.11	15.39%	51.29	11.10%	59.62	12.91%	**49.62**	**10.74%**
9	64.80	14.03%	68.71	14.87%	76.50	16.56%	77.86	16.85%	54.67	11.83%	62.51	13.53%	**49.24**	**10.66%**
10	66.77	14.45%	67.74	14.66%	66.16	14.32%	70.65	15.29%	55.55	12.03%	60.65	13.13%	**52.96**	**11.46%**
Average	66.12	14.31%	67.78	14.67%	67.77	14.67%	71.92	15.57%	53.92	11.67%	59.63	12.91%	**50.00**	**10.82%**
Std dev	1.35		3.85		5.04		2.66		3.92		2.72		**2.71**	

Table 6. RMSEPs from different models for standardized dataset with *19.05 mm* MOR (*kPa*) as the response.

	MLR		PLS		NN		DT		BRT		BSF		**BART**	
	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP
1	733.84	13.00%	741.23	13.13%	694.19	12.29%	823.06	14.58%	670.57	11.88%	585.30	10.37%	**565.59**	**10.02%**
2	740.38	13.11%	740.36	13.11%	692.71	12.27%	776.14	13.74%	649.56	11.50%	616.01	10.91%	**575.98**	**10.20%**
3	747.17	13.23%	746.95	13.23%	735.94	13.03%	826.92	14.64%	658.88	11.67%	594.34	10.53%	**564.04**	**9.99%**
4	739.42	13.09%	738.76	13.08%	729.93	12.93%	874.71	15.49%	660.17	11.69%	**571.25**	**10.12%**	707.32	12.53%
5	766.07	13.57%	773.44	13.70%	734.33	13.00%	776.42	13.75%	644.47	11.41%	**497.70**	**8.81%**	578.69	10.25%
6	723.23	12.81%	756.62	13.40%	742.79	13.15%	813.04	14.40%	659.84	11.69%	624.32	11.06%	**573.67**	**10.16%**
7	740.46	13.11%	742.36	13.15%	757.87	13.42%	815.34	14.44%	648.95	11.49%	630.73	11.17%	**583.03**	**10.32%**
8	731.14	12.95%	756.83	13.40%	719.51	12.74%	862.05	15.27%	651.51	11.54%	646.07	11.44%	**551.98**	**9.78%**
9	744.77	13.19%	762.13	13.50%	776.87	13.76%	714.48	12.65%	645.71	11.43%	726.01	12.86%	**543.03**	**9.62%**
10	736.01	13.03%	741.07	13.12%	771.73	13.67%	881.27	15.61%	660.44	11.70%	649.24	11.50%	**548.30**	**9.71%**
Average	740.25	13.11%	749.98	13.28%	735.59	13.03%	816.34	14.46%	655.01	11.60%	614.10	10.88%	**579.16**	**10.26%**
Std dev	11.39		11.69		28.68		51.06		8.25		59.41		**47.01**	

Table 7. RMSEPs from different models for standardized dataset with *19.05 mm* IB (*kPa*) as the response.

	MLR		PLS		NN		DT		BRT		BSF		**BART**	
	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP	RMSEP	NRMSEP
1	64.98	11.22%	64.97	11.22%	62.97	10.87%	71.89	12.41%	57.12	9.86%	**51.20**	**8.84%**	53.32	9.21%
2	64.87	11.20%	66.24	11.44%	67.10	11.59%	72.06	12.44%	59.14	10.21%	**52.50**	**9.06%**	53.95	9.32%
3	64.91	11.21%	65.79	11.36%	64.40	11.12%	69.20	11.95%	56.63	9.78%	51.66	8.92%	**50.86**	**8.78%**
4	64.70	11.17%	64.53	11.14%	61.55	10.63%	75.49	13.03%	56.77	9.80%	**50.58**	**8.73%**	51.86	8.95%
5	65.66	11.34%	65.67	11.34%	63.93	11.04%	74.24	12.82%	55.89	9.65%	49.92	8.62%	**49.58**	**8.56%**
6	69.98	12.08%	67.88	11.72%	60.02	10.36%	70.76	12.22%	56.73	9.80%	52.84	9.12%	**51.49**	**8.89%**
7	64.16	11.08%	64.53	11.14%	60.92	10.52%	68.69	11.86%	59.49	10.27%	53.34	9.21%	**50.52**	**8.72%**
8	68.80	11.88%	68.45	11.82%	65.31	11.28%	77.96	13.46%	60.80	10.50%	54.21	9.36%	**51.21**	**8.84%**
9	64.10	11.07%	64.37	11.11%	64.12	11.07%	73.79	12.74%	57.03	9.85%	**49.25**	**8.50%**	53.21	9.19%
10	64.21	11.09%	70.06	12.10%	60.34	10.42%	69.70	12.03%	**50.78**	**8.77%**	53.85	9.30%	53.21	9.19%
Average	65.64	11.33%	66.25	11.44%	63.07	10.89%	72.38	12.50%	57.04	9.85%	51.94	8.97%	**51.92**	**8.96%**
Std dev	2.05		1.93		2.32		2.98		2.70		1.69		**1.44**	

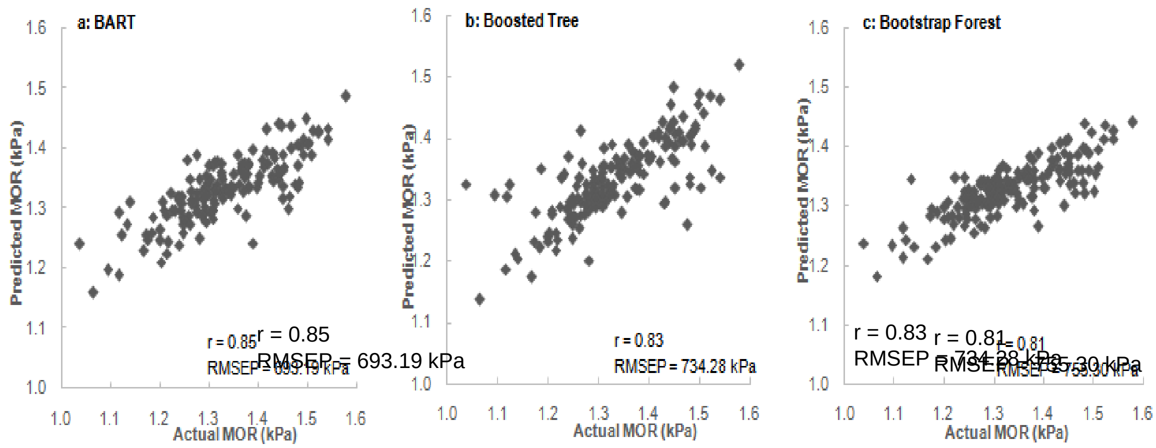


Fig 1. Plots of predicted 12.70 *mm* MOR ($\times 10^4$ *kPa*) versus actual MOR ($\times 10^4$ *kPa*) for the top three best predictive models.

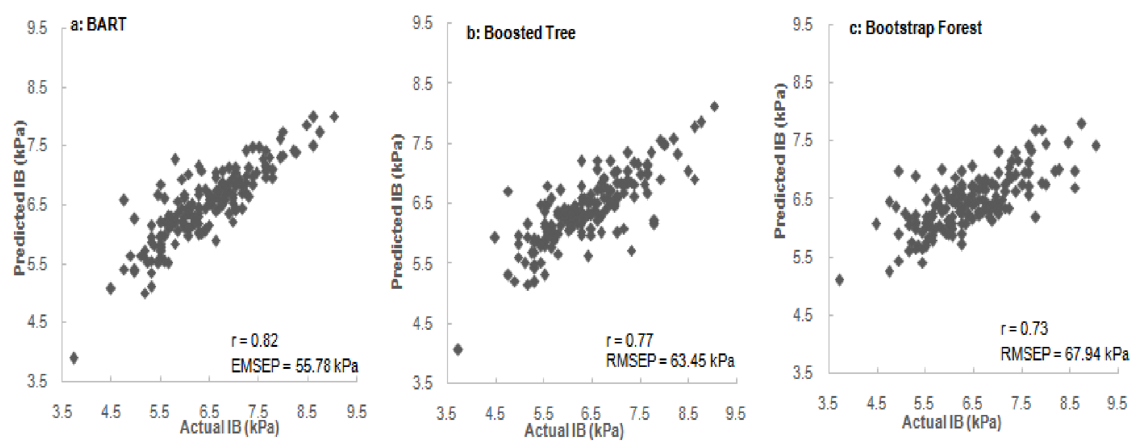


Fig 2. Plots of predicted 12.70 *mm* IB ($\times 10^2$ *kPa*) versus actual IB ($\times 10^2$ *kPa*) for the top three best predictive models.

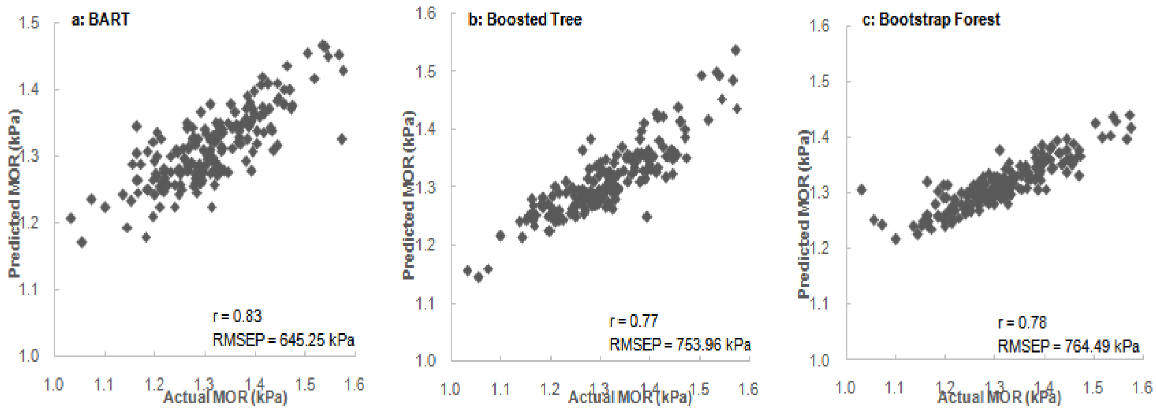


Fig 3. Plots of predicted 15.88 *mm* MOR ($\times 10^4$ *kPa*) versus actual MOR ($\times 10^4$ *kPa*) for the top three best predictive models.

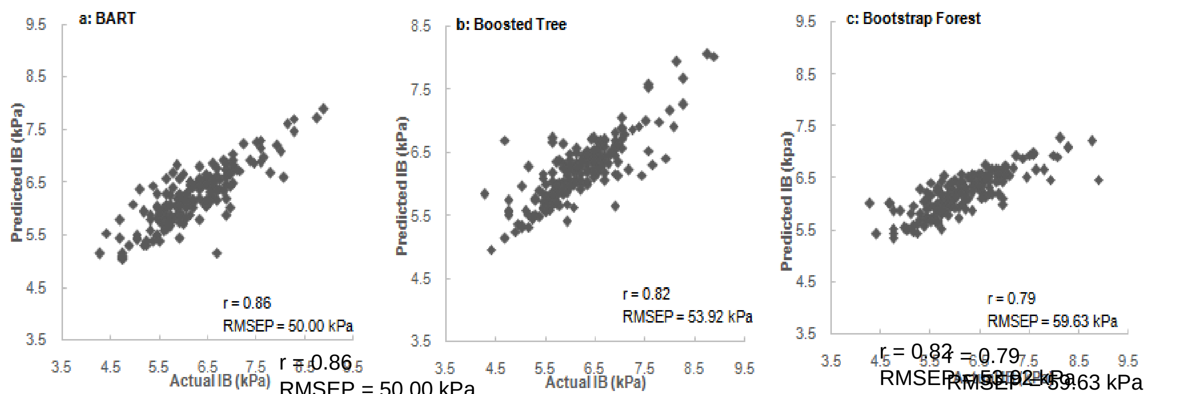


Fig 4. Plots of predicted 15.88 *mm* IB ($\times 10^2$ *kPa*) versus actual IB ($\times 10^2$ *kPa*) for the top three best predictive models.

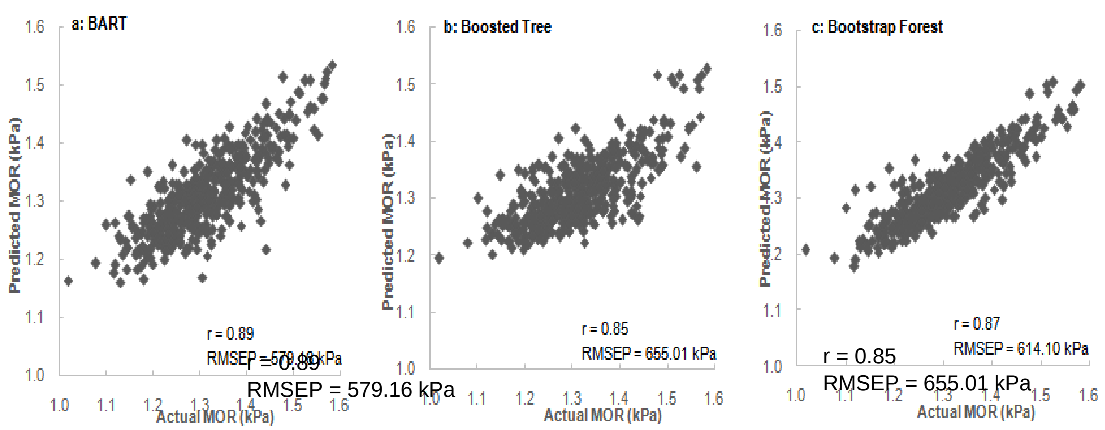
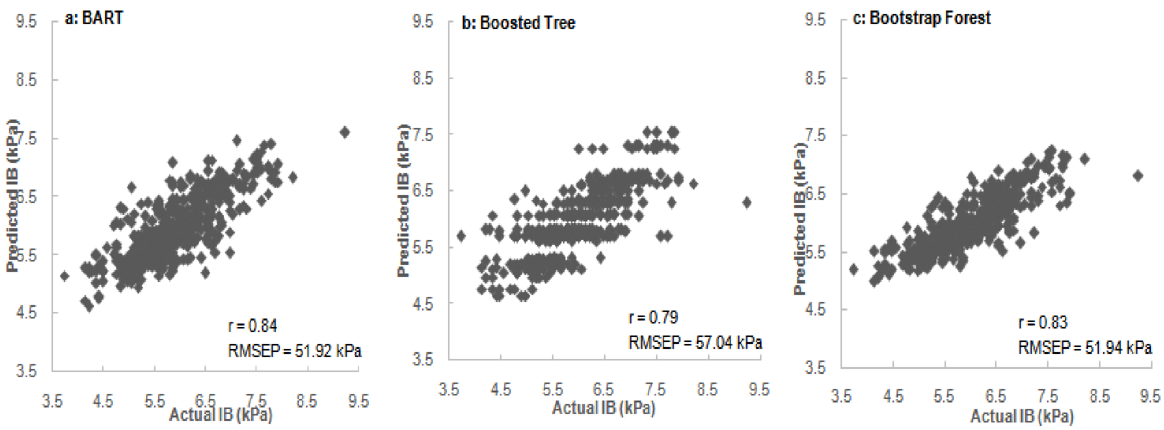


Fig 5. Plots of predicted 19.05 *mm* MOR ($\times 10^4$ *kPa*) versus actual MOR ($\times 10^4$ *kPa*) for the top three best predictive models.



6. Plots of predicted 19.05 mm IB ($\times 10^2$ *kPa*) versus a ctual IB ($\times 10^2$ *kPa*) for the top three best predictive models.

Nana Tian, PhD "Improved Predictive Modeling Using Bayesian Additive Regression Trees (BART) For Wood Composite Products" Research Inventy: International Journal of Engineering And Science, vol. 08, no. 02, 2018, pp. 44–53.