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Traversable Wormhole Solutions in Exponential F(Q) gravity

R.N Patra
Department of Mathematics, Berhampur University, Berhampur, Odisha, India.

Abstract: *This work investigates traversable wormhole solutions in exponential $F(Q)$gravity within the framework of symmetric teleparallel geometry. In this theory, gravitational interaction arises from the non-metricity scalar Q, which characterizes the variation of the metric under parallel transport. A static spherically symmetric wormhole spacetime is considered, and the corresponding non-metricity scalar is explicitly calculated. Using the exponential model $F(Q) = Q + \alpha e^{\beta Q}$, the modified gravitational field equations are derived and solved for an anisotropic matter distribution. The physical properties of the obtained wormhole configuration are analyzed through the behavior of the energy density and pressure components, allowing the investigation of the null and weak energy conditions. Furthermore, the amount of exotic matter required to sustain the wormhole geometry is examined using the volume integral quantifier (VIQ). The results indicate that the violation of the energy conditions is confined to a small region near the wormhole throat and that the total amount of exotic matter required is finite. These findings suggest that exponential corrections in $F(Q)$gravity can support traversable wormhole geometries while reducing the amount of exotic matter needed to maintain the wormhole structure.*

Keywords: *Wormhole, non-metricity, $F(Q)$ gravity, Energy Conditions, Stability Analysis.*

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I. INTRODUCTION

Wormholes are hypothetical spacetime structures that connect two cosmological through a tunnel-like geometry. These solutions arise naturally from the gravitational field equations of General Relativity(GR), where spacetime curvature is determined by the distribution of matter and energy. But nowadays GR has been replaced by Modified gravity. Early theoretical investigations of wormholes were motivated by the possibility of shortcuts between distant points in spacetime and potential applications in interstellar travel. A detailed analysis of traversable wormholes was presented by Morris and Thorne, who demonstrated that a static spherically symmetric spacetime can contain a throat connecting two asymptotically flat regions (Morris and Thorne, 1988). However, such wormhole geometries typically require exotic matter sources that violate the null energy condition, which raises important questions regarding their physical feasibility.

The limitations associated with wormhole solutions in classical gravity have motivated the investigation of modified theories of gravity. Observational evidence such as the accelerated expansion of the Universe suggests that the standard formulation of gravitational interaction may require modification at cosmological scales (Riess et al., 1998; Perlmutter et al., 1999). Various extensions of Einstein's theory have therefore been proposed in which the gravitational action is generalized to include functions of geometric invariants. Among these approaches, curvature-based models such as $f(R)$gravity and torsion-based theories like $f(T)$gravity have attracted considerable attention. These frameworks provide alternative geometric interpretations of gravity and have been applied successfully to cosmology, black hole physics, and astrophysical phenomena (Nojiri and Odintsov, 2011; Cai et al., 2016).

More recently, a new geometric formulation known as F(Q) gravity has been proposed within the framework of symmetric teleparallel gravity (STEGR). In this approach the gravitational interaction is attributed to the non-metricity of spacetime rather than curvature or torsion. The fundamental quantity describing this geometry is the non-metricity tensor, which measures the variation of the metric under parallel transport. The dynamics of the theory are determined by the non-metricity scalar Q, and the gravitational action is generalized to an arbitrary function $F(Q)$. This formulation has attracted growing interest because it provides a consistent geometric extension of gravity and has been successfully applied to several cosmological and astrophysical problems (Jiménez et al., 2018).

An important aspect in the study of wormhole geometries is the analysis of energy conditions. These conditions place restrictions on the energy–momentum tensor and are widely used to determine whether physically reasonable matter sources can support a given spacetime configuration. In the context of wormholes, the null energy condition (NEC) plays a particularly significant role, since its violation indicates the existence of exotic matter in General Relativity or modified gravity. Modified gravity theories may alter the effective gravitational field equations in such a way that the geometric contributions themselves can account for the required energy condition violations, thereby reducing or eliminating the need for exotic matter sources (Visser, 1995; Lobo, 2005).

In addition to the analysis of energy conditions, it is also important to estimate the amount of exotic matter required to sustain wormhole configurations. This aspect can be examined using the Volume Integral Quantifier (VIQ), which provides a measure of the total quantity of matter that violates the null energy condition within the wormhole spacetime. The VIQ allows one to determine whether the exotic matter required to support the wormhole geometry is finite and localized near the throat. If the integral remains small, the wormhole solution can be considered physically more acceptable. Motivated by these considerations, the present work investigates traversable wormhole solutions within the framework of exponential $F(Q)$gravity, focusing on the geometric structure of spacetime, the behavior of energy conditions, and the amount of exotic matter required to sustain the resulting wormhole configuration.

II. WORMHOLE FIELD EQUATIONS IN F(Q) GRAVITY

2.1 **Wormhole Metric.**

We consider the static spherically symmetric Morris–Thorne wormhole metric (Morris and Thorne, 1988)

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1-b(r)/r} + r^2(d\theta^2 + sin^2\theta d\phi^2) \quad (1)$$

where

Φ(r) is the redshift function and b(r) is the shape function.

The matter threading the wormhole is assumed to be **anisotropic fluid**, whose energy–momentum tensor is

$$T^{\mu}_{\nu} = diag(-\rho, p_r, p_t, p_t) \quad (2)$$

where

- ρ= energy density
- p_r= radial pressure
- p_t= tangential pressure.

The wormhole throat occurs at $r = r0$ such that $b(r0) = r0$
The flare-out condition requires $b'(r0) < 1$

and asymptotic flatness requires $\frac{b(r)}{r} \to 0, (r \to \infty)$. The redshift function must remain finite everywhere to avoid event horizons.

We consider the power-law shape function

$$b(r) = r_0 \left(\frac{r_0}{r}\right)^n \quad (3)$$

where $n > 0$.
The derivative becomes

$$b'(r) = -n\, r_0^{n+1} r^{-(n+1)} \quad (4)$$

At the throat

$$b'(r_0) = -\frac{n}{r_0} < 1 \quad (5)$$

which satisfies the flare-out condition.
The action of $F(Q)$gravity is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(Q) + L_m\right] \quad (6)$$

Variation of the action gives the modified field equations

$$\frac{2}{\sqrt{-g}}\partial_\alpha\left(\sqrt{-g}F_Q P^{\alpha}_{\mu\nu}\right) + \frac{1}{2}g_{\mu\nu}F + F_Q\left(P_{\mu\alpha\beta}Q_{\nu}^{\alpha\beta}\right) = T_{\mu\nu} \quad (7)$$

where

$$F_Q = \frac{dF}{dQ} \tag{8}$$

In the coincident gauge

$$\Gamma^{\alpha}_{\mu\nu} = 0$$

therefore

$$Q_{\alpha\mu\nu} = \partial_\alpha g_{\mu\nu} \tag{9}$$

The traces of the non-metricity tensor are

$$Q_\alpha = Q_{\alpha\ \mu}^{\ \mu}$$
$$\tilde{Q}_\alpha = Q^{\mu}_{\ \alpha\mu} \tag{10}$$

For the wormhole metric these reduce to

$$Q_r = \frac{-b'(r)/r + b(r)/r^2}{1 - \frac{b(r)}{r}} + \frac{4}{r}$$

and

$$\tilde{Q}_r = -\frac{-b'(r)/r + b(r)/r^2}{1 - \frac{b(r)}{r}} \tag{11}$$

The scalar is defined as

$$Q = -Q_{\alpha\mu\nu} P^{\alpha\mu\nu} \tag{12}$$

Substituting the wormhole metric gives

$$Q(r) = \frac{2}{r^2}\left(1 - \frac{b(r)}{r}\right)\left(\frac{b(r)}{r}\right) \tag{13}$$

Using the chosen shape function

$$b(r) = r_0^{n+1} r^{-n}$$

the scalar becomes

$$Q(r) = \frac{2}{r^2}\left(1 - \frac{r_0^{n+1}}{r^{n+1}}\right)\left(\frac{r_0^{n+1}}{r^{n+1}}\right) \tag{14}$$

We consider

$$F(Q) = Q + \alpha e^{\beta Q} \tag{15}$$

The derivative becomes

$$F_Q = 1 + \alpha\beta e^{\beta Q} \tag{16}$$

The matter components are obtained as

Energy Density

$$\rho = \frac{F}{2} + \frac{F_Q}{r^2} b'(r) \tag{17}$$

Radial Pressure

$$p_r = -\frac{F}{2} + F_Q \frac{b(r)}{r^3} \tag{18}$$

Tangential Pressure

$$p_t = -\frac{F}{2} + F_Q \frac{b'(r) - \frac{b(r)}{r}}{2r^2} \tag{19}$$

Substituting $b(r)$, $b'(r)$, $F(Q)$, and F_Q the matter components become

$$\rho = \frac{1}{2}\left(Q + \alpha e^{\beta Q}\right) - \frac{n r_0^{n+1}}{r^{n+3}}\left(1 + \alpha\beta e^{\beta Q}\right)$$

$$p_r = -\frac{1}{2}\left(Q + \alpha e^{\beta Q}\right) + \frac{r_0^{n+1}}{r^{n+3}}\left(1 + \alpha\beta e^{\beta Q}\right)$$

$$p_t = -\frac{1}{2}\left(Q+\alpha e^{\beta Q}\right) + \frac{-nr_0^{n+1}r^{-(n+1)} - \frac{r_0^{n+1}}{r^{n+1}}}{2r^2}\left(1+\alpha\beta e^{\beta Q}\right)$$

To obtain fully explicit expressions, we now substitute the non-metricity scalar $Q(r)$into the matter components.

Which gives

$$\rho = \frac{1}{2}\left[\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right) + \alpha\exp\left(\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)\right)\right] - \frac{nr_0^{n+1}}{r^{n+3}}\left[1+\alpha\beta\exp\left(\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)\right)\right]$$

$$p_r = -\frac{1}{2}\left[\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right) + \alpha\exp\left(\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)\right)\right] + \frac{r_0^{n+1}}{r^{n+3}}\left[1+\alpha\beta\exp\left(\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)\right)\right]$$

$$p_t = -\frac{1}{2}\left[\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right) + \alpha\exp\left(\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)\right) + \frac{-nr_0^{n+1}r^{-(n+1)} - \frac{r_0^{n+1}}{r^{n+1}}}{2r^2}\left[1+\alpha\beta\exp\left(\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)\right)\right]\right] \tag{20}$$

2.2 **Energy Conditions**

Energy conditions provide important criteria for determining whether the matter distribution supporting the wormhole geometry is physically reasonable. These conditions impose constraints on the energy–momentum tensor $T_{\mu\nu}$.

Null Energy Condition (NEC)

The null energy condition requires that

$$\rho + p_r \geq 0$$

and

$$\rho + p_t \geq 0 \tag{21}$$

By Using the expressions (17),(18),(19) and (21) gives

$$\rho + p_r = \frac{F_Q}{r^2}b'(r) + \frac{F_Q}{r^3}b(r)$$

and

$$\rho + p_t = \frac{F_Q}{r^2}b'(r) + \frac{F_Q}{2r^2}\left(b'(r) - \frac{b(r)}{r}\right)$$

Simplifying the above relation using (20) leads to

$$\rho + p_r = \frac{(1-n)r_0^{n+1}}{r^{n+3}}\left(1+\alpha\beta e^{\beta Q}\right)$$

and

$$\rho + p_t = -\frac{(n+1)r_0^{n+1}}{2r^{n+3}}\left(1+\alpha\beta e^{\beta Q}\right) \tag{22}$$

Weak Energy Condition (WEC)

The weak energy condition requires

$$\rho \geq 0$$

together with the NEC conditions

$$\rho + p_r \geq 0, \rho + p_t \geq 0$$

Substituting the energy density expression

$$\rho = \frac{1}{2}\left(Q + \alpha e^{\beta Q}\right) - \frac{n r_0^{n+1}}{r^{n+3}}\left(1 + \alpha\beta e^{\beta Q}\right)$$

allows one to analyse the regions where the WEC is satisfied.
Simplifying the above relation using (20) leads to

$$\rho = \frac{1}{2}\left[\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right) + \alpha e^{\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)}\right] - \frac{n r_0^{n+1}}{r^{n+3}}\left[1+\alpha\beta e^{\beta\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right)}\right] \quad (23)$$

Strong Energy Condition (SEC)
For anisotropic matter the strong energy condition is written as

$$\rho + p_r + 2p_t \geq 0$$

Substituting the obtained expressions for the pressures gives

$$\rho + p_r + 2p_t = -\left(Q+\alpha e^{\beta Q}\right) + F_Q\left[\frac{2b'(r)}{r^2} + \frac{b(r)}{r^3} - \frac{b(r)}{r^3}\right]$$

Simplifying the above relation leads to

$$\rho + p_r + 2p_t = -\left(Q+\alpha e^{\beta Q}\right) + \frac{2F_Q b'(r)}{r^2}$$

Simplifying the above expression. Using (20) gives

$$\rho + p_r + 2p_t = -\left[\frac{2r_0^{n+1}}{r^{n+3}}\left(1-\frac{r_0^{n+1}}{r^{n+1}}\right) + \alpha e^{\beta Q}\right] - \frac{2n r_0^{n+1}}{r^{n+3}}\left(1+\alpha\beta e^{\beta Q}\right) \quad (24)$$

Evaluation at the Wormhole Throat
At the throat $r = r_0$

$$b(r_0) = r_0$$
$$b'(r_0) < 1.$$

Substituting these relations into the NEC expression gives

$$(\rho + p_r)_{r=r_0} = \frac{F_Q}{r_0^2}\left(b'(r_0) + 1\right)$$

Thus the violation or satisfaction of the energy conditions depends on the parameters $\alpha$and $\beta$of the exponential $F(Q)$model.

2.3 **Stability Analysis**

The violation of the null energy condition near the wormhole throat indicates the presence of exotic matter that is required to sustain the wormhole geometry. However, an important question concerns the total amount of such exotic matter present in the spacetime. To address this issue, the **Volume Integral Quantifier (VIQ)** is employed, which provides a measure of the integrated energy-condition violation within the wormhole spacetime (Visser, 1995; Visser et al., 2003).
The volume integral quantifier is defined as

$$I_V = \int (\rho + p_r) dV \quad (25)$$

where ρrepresents the energy density and $p_r$denotes the radial pressure. For a spherically symmetric wormhole spacetime, the volume element can be written as

$$dV = 4\pi r^2 dr.$$

Substituting this expression into the integral, the volume integral quantifier becomes

$$I_V = 4\pi \int_{r_0}^{\infty} (\rho + p_r) r^2 dr, \quad (26)$$

where $r_0$is the radius of the wormhole throat.
The quantity $I_V$determines the total amount of exotic matter required to support the wormhole structure. If the integral remains finite and small, the wormhole configuration can be considered physically more viable since the violation of the energy conditions is confined to a limited region around the throat. Therefore, the evaluation of

the volume integral quantifier plays an important role in assessing the physical acceptability of wormhole solutions in modified gravity theories.

III. RESULTS AND DISCUSSIONS

In this section we discuss the physical behaviour of the wormhole solution obtained in the framework of exponential $F(Q)$gravity. The analysis is performed using the expressions derived for the energy density and pressures in terms of the chosen shape function and model parameters.

3.1. Energy Conditions

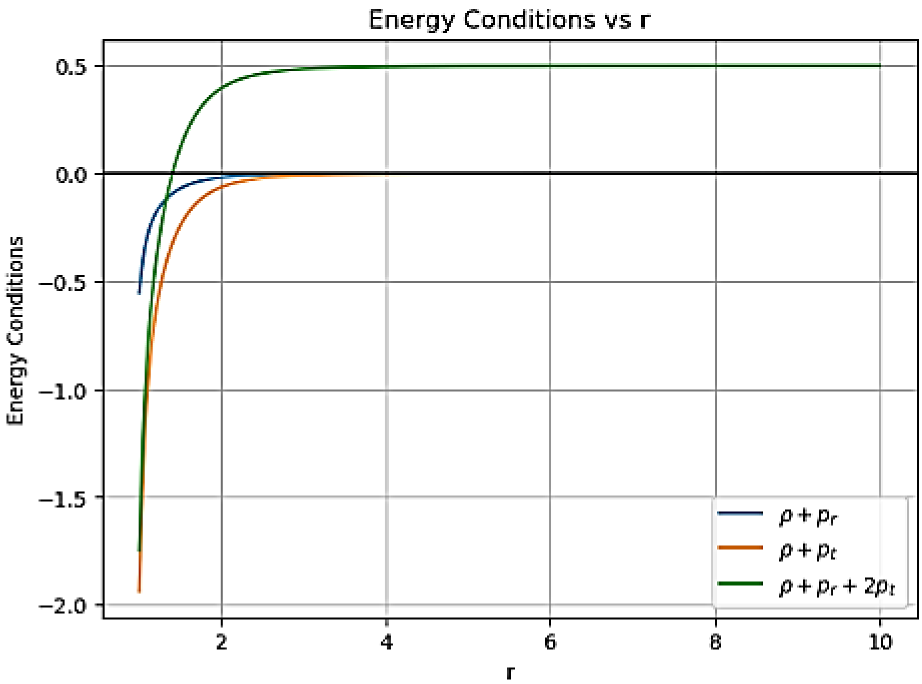
The behaviour of the energy conditions is illustrated in **Fig. 1**, which shows the radial null energy condition $\rho + p_r$, the transverse null energy condition $\rho + p_t$, and the strong energy condition $\rho + p_r + 2p_t$as functions of the radial coordinate r.

From the figure it can be observed that both $\rho + p_r$and $\rho + p_t$are negative in the vicinity of the wormhole throat and gradually approach zero as the radial coordinate increases. This indicates that the **null energy condition (NEC) is violated only in a small region near the throat**, which is a typical feature of traversable wormhole geometries.

Such a violation implies the presence of exotic matter required to support the wormhole structure. However, the fact that the violation is localized near the throat suggests that the amount of exotic matter required is limited.

On the other hand, the quantity $\rho + p_r + 2p_t$, associated with the **strong energy condition (SEC)**, becomes positive away from the throat, indicating that the spacetime behaves normally at larger radial distances.

Figure-1. Plotting of Energy Conditions.



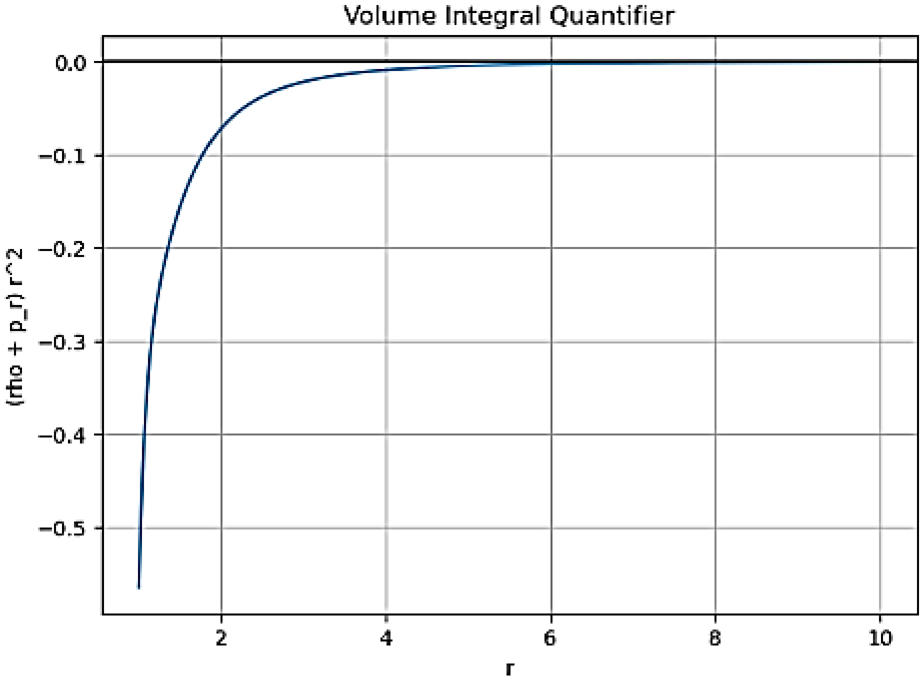
3.2. Volume Integral Quantifier

To further investigate the amount of exotic matter required to sustain the wormhole geometry, we compute the Volume Integral Quantifier (VIQ), defined as

$$I_V = 4\pi \int_{r_0}^{\infty} (\rho + p_r) r^2 \, dr. \tag{27}$$

This integral provides a measure of the total amount of exotic matter present in the wormhole spacetime.
The behaviour of the integrand $(\rho + p_r)r^2$is shown in **Fig. 2**.

Figure-2. Plotting of Energy Conditions.



It can be seen that the integrand is negative near the wormhole throat and rapidly approaches zero as $r$increases. This confirms that the violation of the null energy condition is **confined to a small region near the throat**. The numerical value of the volume integral quantifier obtained from the analysis is

$$I_V = -3.2757524253234345$$

The negative value of $I_V$confirms the presence of exotic matter, while its finite magnitude indicates that **only a limited amount of exotic matter is required to sustain the wormhole configuration**.

For clarity, the behaviour of the energy conditions at the wormhole throat is summarized in **Table 1**.

Table-1Characterization data through the fragmentation profile in mass spectrometer and retention time of the derivatives of ibuprofen.

Energy Condition	Mathematical Form	Result
Radial NEC	$\rho+p_r\geq0$	Violated near throat
Transverse NEC	$\rho+p_t\geq0$	Violated near throat
WEC	$\rho\geq0$	Partially satisfied
SEC	$\rho+p_r+2p_t\geq0$	Satisfied away from throat

3.3 Physical Interpretation.

The obtained results demonstrate that the exponential $F(Q)$gravity model can support traversable wormhole geometries with **localized violations of the energy conditions**. The violation of the NEC is confined to the vicinity of the throat, while the spacetime behaves normally far from the wormhole. Furthermore, the finite value of the volume integral quantifier suggests that the total amount of exotic matter required to maintain the wormhole configuration is relatively small.

These features indicate that modified gravity theories such as $F(Q)$gravity provide a viable framework for constructing wormhole solutions with physically acceptable properties.

IV. CONCLUSION

In this work, we investigated static spherically symmetric wormhole solutions in the framework of exponential $F(Q)$gravity. By considering a suitable wormhole metric with a specific choice of shape function, the modified field equations were derived and the corresponding energy density and pressure components were obtained. The physical properties of the wormhole geometry were analyzed through the study of energy conditions and the volume integral quantifier.

The behaviour of the energy conditions indicates that the null energy condition is violated in the vicinity of the wormhole throat, which is a necessary requirement for sustaining traversable wormhole geometries. However, the violation is confined to a small region near the throat, while the energy-condition quantities approach zero as the radial coordinate increases. This suggests that the exotic matter required to maintain the wormhole structure is localized around the throat.

Furthermore, the amount of exotic matter present in the spacetime was quantified using the volume integral quantifier. The obtained results show that the integral is finite and relatively small, indicating that only a limited amount of exotic matter is required to support the wormhole configuration in the considered exponential $F(Q)$gravity model.

Overall, the results demonstrate that modified gravity theories based on non-metricity, such as $F(Q)$gravity, provide a viable framework for the construction of traversable wormhole solutions with physically acceptable properties. The exponential $F(Q)$model considered in this work allows the existence of wormhole geometries where the violation of energy conditions is localized near the throat, thereby reducing the amount of exotic matter required.

Future investigations may explore more general shape functions and alternative $F(Q)$models to further understand the role of non-metricity in supporting wormhole geometries.

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